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Dynamics of a vortex line in type-II superconductors

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Abstract. We numerically study the dynamics of a vortex line in type-II superconductors which is driven by vortex line tension, external driving force, pinning force and thermal fluctuations. Based on the local stochastic differential equation for the vortex motion, computer simulations are performed within the planar approximation (no torsion) to investigate the pinning time, which is the time for the vortex line to overcome the pinning centre.

The dynamics of topological defects such as domain walls, interfaces, and vortices have attracted much attention from the point of view of pattern formation [1]. Of various topological defects, vortex line (VL) structures are widely seen in many areas of physics, e.g. magnetic flux in type-II superconductors [2], VL in a superfluid [3], disclination in liquid crystals [4], dislocation in solids [5] and cosmic string in the early universe [6]. Recently, the thermal behaviour of the VL lattice in high-temperature superconductors has also been found to exhibit fascinating phenomena [7].

As the first step of our work to discuss such vortex motions, we here study the dynamical behaviour of VL in type-II superconductors which interacts with an impurity (pinning centre) and random thermal forces. Although the behaviour of only one VL is sufficiently far removed from reality, this situation is chosen as suitable for studying an elementary process of the VL motion.

Two-dimensional simulations of thermal behaviour of the VL lattice interacting with random impurities have been done in [8–10], where the VLs (taken to be in the z direction) are projected onto the x–y plane and then the VL lattice is regarded as an ensemble of interacting particles on the plane. Therefore, in their simulations, effects in the z direction such as a line tension have been neglected. On the other hand, on the basis of the two-dimensional time-dependent Ginzburg–Landau equations we have investigated pattern formation of the magnetic flux in a superconducting film [11]. Thus, the present work is in some manner complementary to those studies in the sense that we take into account effects of the vortex line tension as well as the transport current.

Now we consider a flat plate of type-II superconductor in the x–y plane with thickness d in the presence of the external magnetic field applied along the z axis. The transport current flows in the y direction. Then, once we consider a single VL with its torsion being zero; the following motion of the VL is restricted to the x–z plane in the absence of the thermal fluctuations. Thus, for simplicity we assume that the VL profile is defined in the two-dimensional x–z plane even in the presence of thermal fluctuations (the planar approximation) and is given by a function $x = h(z, t)$ at time t with its normal unit vector

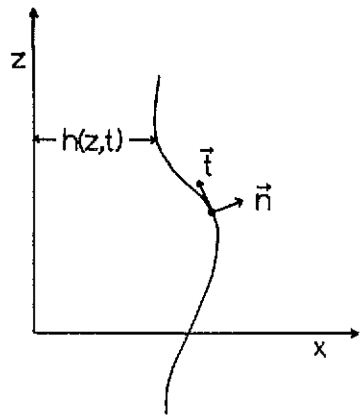

Figure 1. Geometry and notations of the VL under consideration. See text.

$\boldsymbol{n}$ and tangent unit vector $\boldsymbol{t}$, as is shown in figure 1. By neglecting the overhangs of the VL, $h(z, t)$ is taken to be a single-valued function.

In the above situation the equation of motion of the VL can be described by [12, 13]

$$\Gamma^{-1}(\partial \boldsymbol{R}/\partial t) = [\varepsilon(T)K + (\Phi_0/c)/j]\boldsymbol{n} - (\partial/\partial \boldsymbol{R})[V(\boldsymbol{R}, \boldsymbol{R}_p)] + f(z, t) \qquad (1)$$

where $\boldsymbol{R} \equiv (h(z, t), z)$ denotes the position vector of the VL in the x–z plane with its curvature K, its normal unit vector $\boldsymbol{n}$ and line tension $\varepsilon(T) \equiv [\Phi_0/4\pi\lambda(T)]^2 \ln \kappa$ at temperature T, j is the transport current density, Φ_0 is the flux quantum and c is the velocity of light. Here $\Gamma^{-1} \equiv \sigma B\Phi_0/c^2$ is the kinetic coefficient with the normal conductivity σ and the magnetic field B, and $\kappa \equiv \lambda(T)/\xi(T)$ is the Ginzburg–Landau (GL) parameter. We assume that at all temperatures the coherent length $\xi(T)$ and the magnetic penetration depth $\lambda(T)$ are defined by $\xi(T) \equiv \xi(0)(1 - T/T_c)^{-1/2}$ and $\lambda(T) \equiv \lambda(0)(1 - T/T_c)^{-1/2}$, respectively, with the critical temperature T_c, so that the GL parameter κ becomes independent of temperature, i.e. $\kappa \equiv \lambda(0)/\xi(0)$. The potential energy of the VL due to an impurity located at $\boldsymbol{R}_p$ is given by [14]

$$V(\boldsymbol{R}, \boldsymbol{R}_p) \equiv -A \exp(-|\boldsymbol{R} - \boldsymbol{R}_p|^2/a^2) \qquad (2)$$

with

$$A \equiv \alpha H_{c2}(T)^2[1 - B/H_{c2}(T)][\pi\xi(T)^2/16\pi\kappa^2] \qquad (3)$$

where α $(0 \leqslant \alpha \leqslant 1)$ in equation (3) denotes a fraction of the condensation energy stored per length in a cylinder of the size of the VL core, a in equation (2) is the range of the pinning centre, and $H_{c2}(T)$ is the upper critical field. Finally, the last term in equation (1) describes the thermodynamic random force with zero mean and the correlation given by [12, 13]

$$\langle f_i(z, t(f_j(z', t')\rangle = 2k_B T\Gamma\delta_{ij}\delta(z - z')\delta(t - t') \qquad (4)$$

where f_i denotes the ith component of f and $k_B T$ is the usual thermal energy.

In the following we concentrate only on the normal motion of the VL, since the main two driving forces in equation (1), $\varepsilon(T)K + (\Phi_0/c)j$, are proportional to $\boldsymbol{n}$ [1]. Moreover, it is convenient to introduce the scaling such that length and time are measured in units of $\xi(0)$ and $t_0 \equiv \pi\hbar/96k_BT_c$, respectively. In these units the normal motion of the VL can be written by

$$n \cdot \partial R/\partial t = C_1 K + C_2(T) - C_3 n \cdot (R - R_p) \exp(-|R - R_p|^2) + C_4(T) F(z, t) \tag{5}$$

with

$$C_1 \equiv \kappa^2 \tag{6}$$

$$C_2(T) \equiv 4\sqrt{3}\kappa^2/(9 \ln \kappa)(1 - T/T_c)^{-1}(j/j_0) \tag{7}$$

$$C_3 \equiv \alpha[\kappa^2/(2 \ln \kappa)][1 - (\ln \kappa)/2\kappa^2] \tag{8}$$

$$C_4(T) \equiv \sqrt{2k_B T_c \kappa^3/\lambda(0)\varepsilon(0)}\,(T/T_c)^{1/2}\,(1 - T/T_c)^{-1/2} \tag{9}$$

$$\langle F(z,t)F(z',t')\rangle = \delta(z - z')\delta(t - t') \tag{10}$$

where $j_0 \equiv cH_c(0)/3\sqrt{6}\pi\lambda(0)$ with zero-temperature critical field $H_c(0)$ denotes the critical current density associated with thin-film superconductors [15]. In order to obtain the above equations, we have set $B = H_{c1}(T)$, since we have neglected vortex–vortex interactions. Moreover, we have used the phenomenological relations such as $H_{c1}(T) = [(\ln \kappa)/\sqrt{2}\kappa]H_c(T)$, $H_{c2}(T) = \sqrt{2}\kappa H_c(T)$ and $H_c(T) = \Phi_0/2\pi\sqrt{2}\xi(T)\lambda(T)$ [15]. Here we should remark that the temperature effects enter not only through the stochastic term $C_4(T)F(z,t)$ but also through the parameter $C_2(T)$.

The important energy scales in the problem are the line tension $\varepsilon(T)$ and the thermal energy k_BT. We consider a system with $k_BT_c/\varepsilon(0)d = 5 \times 10^{-3}/\ln \kappa$, $d = 50\xi(0)$ and $\kappa = 2$. We also set $a = \xi(0)$ and $R_p = (0, 0)$. This choice of parameters enables us to solve the stochastic equation with reasonable speed and gives a model for the study of fundamental properties of the VL motion, although the parameter set does not directly describe a specific material.

The outline of the procedure of the present simulation is as follows. A VL expressed in Cartesian coordinates by

$$R(s,t) = (x(s,t), z(s,t)) \qquad (0 \leqslant s \leqslant L) \tag{11}$$

with the natural coordinate s along the VL and the total length L of the VL is discretized as

$$R_i(t) = (x(s_i,t), z(s_i,t)) \equiv (x_i(t), z_i(t)) \tag{12}$$

with $s_i \equiv (L/N)i$ $(0 \leqslant i \leqslant N)$, $N = [L/\Delta L]$ and $\Delta L = \xi(0)/4$. Here $[L/\Delta L]$ denotes the nearest integer to $L/\Delta L$. The curvature K_i and the normal unit vector n_i at $s = s_i$ can be approximated, respectively, by [16]

$$K_i = \xi(0)^{-1}(\Delta L_i)^{-2}(\Delta x_i\,\Delta^2 z_i - \Delta^2 x_i\,\Delta z_i) \tag{13}$$

$$n_i = (\Delta L_i)^{-1}(-\Delta z_i, \Delta x_i) \tag{14}$$

with

$$\Delta L_i = [(\Delta x_i)^2 + (\Delta z_i)^2]^{1/2} \tag{15}$$

$$\Delta x_i = (x_{i+1} - x_{i-1})/2 \tag{16}$$

$$\Delta^2 x_i = x_{i+1} + x_{i-1} - 2x_i, \quad \text{etc.} \tag{17}$$

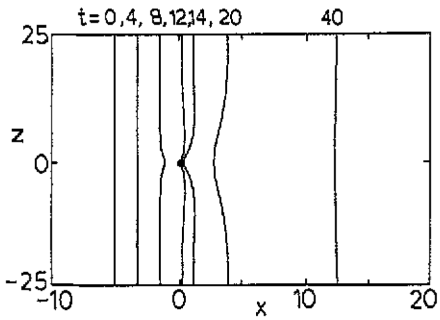

Figure 2. VL motion at $T = 0$ for $\alpha = 1$ and $j/j_0 = 0.1$. The full circle denotes a pinning centre.

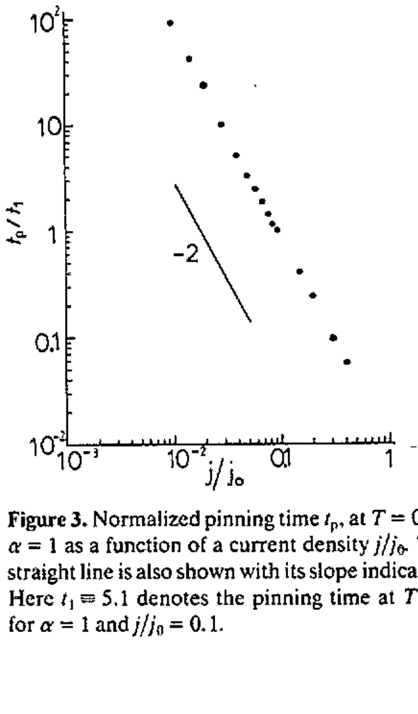

Figure 3. Normalized pinning time t_p, at $T = 0$ for $\alpha = 1$ as a function of a current density j/j_0. The straight line is also shown with its slope indicated. Here $t_1 \equiv 5.1$ denotes the pinning time at $T = 0$ for $\alpha = 1$ and $j/j_0 = 0.1$.

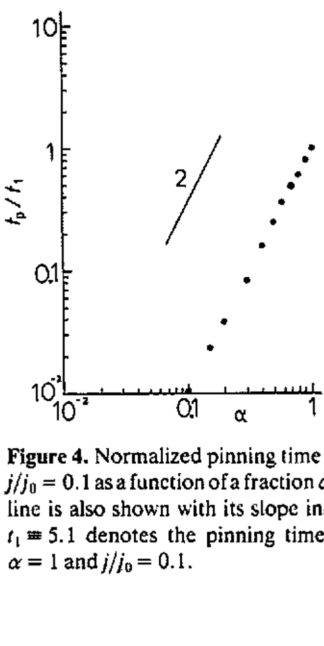

Figure 4. Normalized pinning time t_p at $T = 0$ for $j/j_0 = 0.1$ as a function of a fraction α. The straight line is also shown with its slope indicated. Here $t_1 \equiv 5.1$ denotes the pinning time at $T = 0$ for $\alpha = 1$ and $j/j_0 = 0.1$.

In the actual simulations, the VL is made smooth by using the third-order spline function after every ten time steps [17]. Then, the number N of points is redefined to keep the distance between the dividing points equal. The time step Δt is chosen to satisfy with the condition $\Delta t \max\{C_1, C_2, C_3, C_4\} \leqslant 0.001$. We also impose boundary conditions such that $\boldsymbol{R}_{N+1}(t) = \boldsymbol{R}_N(t)$ and $\boldsymbol{R}_{-1}(t) = \boldsymbol{R}_0(t)$. These boundary conditions mean that the magnetic flux is assumed to be parallel to the z axis outside the sample. The random variable $F(z, t)$ at each site is independently selected from a Gaussian distribution with zero mean and standard deviation $(2\,\Delta t)^{-1/2}$ [18].

In figure 2 we show the time-dependent behaviour of the VL from the initially straight line at $T = 0$ for $\alpha = 1$ and $j/j_0 = 0.1$. Pinning and depinning behaviour can be seen in this figure.

We study the pinning behaviour in a little more detail to estimate a time interval t_p, which denotes the time for the VL to escape from an impurity after being trapped by it (called a pinning time). In figures 3 and 4, we show the pinning time at $T = 0$, as a

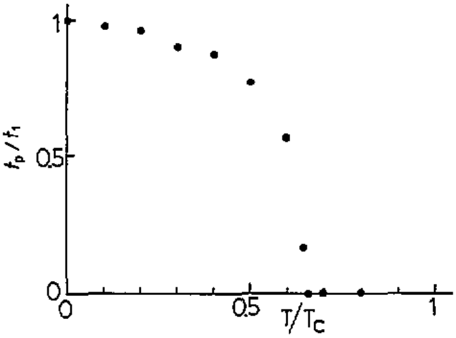

Figure 5. Normalized pinning time t_p, for $\alpha = 1$ and $j/j_0 = 0.1$, as a function of temperature T. Here $t_1 \equiv 5.1$. denotes the pinning time at $T = 0$ for $\alpha = 1$ and $j/j_0 = 0.1$.

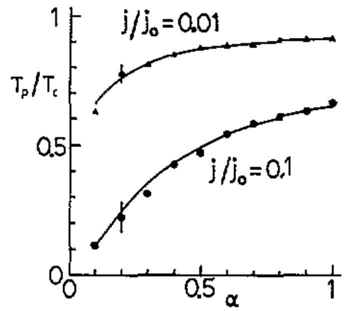

Figure 6. Depinning temperature T_p as a function of α for $j/j_0 = 0.1$ (●) and $j/j_0 = 0.01$ (▲). The bars indicate typical data scatterers. The full curves denote equation (19) with $\beta = 2.48$.

function of the current density j/j_0 and the fraction α, respectively. From these figures we find that the pinning time t_p at $T = 0$ has the relation

$$t_p \propto \alpha^2 (j/j_0)^{-2}. \tag{18}$$

We have numerically checked that the above is unchanged even for the initially random lines. This result is understood as follows. Recalling that approximately we have $\boldsymbol{n} \cdot \partial \boldsymbol{R}/\partial t \simeq \partial h/\partial t$ and $K \simeq \partial^2 h/\partial z^2$ [19], We can regard this model as a diffusion equation with diffusion constant C_1, biased velocity $C_2(0)$ and pinning strength C_3. Thus, the pinning time is estimated to be the diffusion time for a VL to pass an effective range of a pinning centre. Following [19], we find that such effective region is proportional to $C_3/C_2(0)$. Therefore, we obtain the relation $t_p \propto (1/C_1)[C_3/C_2(0)]^2$ and thus equation (18).

Finally, we study the temperature effect on the above results. In figure 5, the pinning time is shown as a function of temperature for the case with $\alpha = 1$ and $j/j_0 = 0.1$. We find that the pinning time rapidly falls to be zero at a certain temperature T_p, below T_c. This result suggests that the thermal behaviour of the system under the transport current changes at $T \simeq T_p$. In figure 6, the depinning temperature T_p, at which the pinning time becomes zero, is shown as a function of α for $j/j_0 = 0.1$ and 0.01. These results are obtained by averaging over 50 independent simulation runs. These depinning phenomena occur when the magnitude of the thermal fluctuation $C_4(T)$ is comparable with the effective pinning region $C_3/C_2(T)$. Thus, we obtain the following relation:

$$\alpha(1 - T_p/T_c)^{3/2} = \beta(j/j_0)(T_p/T_c)^{1/2} \tag{19}$$

with a positive constant β. In figure 6, we have also plotted the relation (19) with $\beta = 2.48$ as full curves for comparison. We can see that the relation (19) agrees well with numerical results. We have numerically checked that the relation (19) as well as (18) are unchanged for $0.1 \leqslant \alpha \leqslant 1$ and $0.01 \leqslant j/j_0 \leqslant 1$, except for the irrelevant coefficient like β, even when the choice of parameters set is changed. Finally, we comment on the planar approximation used above. Off-plane fluctuations neglected here could grow and become important, especially as the line is dragged across the impurity. Thus, we guess the present results should not be taken too literally. Such effects are now under consideration.

In summary, on the basis of stochastic equation of motion for a single planar VL, we have carried out several simulations of the dynamical behaviour of a VL, especially the pinning time. In this stochastic equation, VL tension, external transport current, a pinning centre and thermal fluctuations are taken into account. We have found that at zero temperature the pinning time t_p is satisfied by the relation (18) and that there exists a depinning temperature above which a VL is free from a pinning centre because of the thermal fluctuations. To our knowledge, these behaviours, however, have not yet been reported experimentally. A systematic experimental study of the temperature- and/or the time-dependent behaviour for the pinning is thus highly desirable. At the present stage, it is difficult to guess the dynamical behaviour of the VL lattice in random media since only the dynamics of a single VL have been presented. To do so, we must take into account other effects neglected here, such as vortex–vortex interactions, off-plane fluctuations and detailed temperature dependence of superconducting parameters as well as random impurities. Moreover, to have confidence in the stochastic equation (5) we need to discuss the corresponding Fokker–Planck equation, and to solve it in simple cases. These problems are interesting and still remain open. However, we expect that the present method of research will provide us with a useful tool for studying such effects and also the essence of the dynamics of other topological defects.

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